German University in Cairo
Department of Computer Science
Assoc. Prof. Haythem O. Ismail

Analysis and Design of Algorithms, Winter 2017
Assignment 1

Exercise 1-1

Prove that Θ is an equivalence relation.

Exercise 1-2

Asymptotically rank the following functions.
n, $n^{1/2}$, $\log(n)$, $\log(\log(n))$, $\log^2 n$, $(1/3)^n$, 4, $(3/2)^n$, $n!$.

Exercise 1-3

Prove that, for $a, b \in \mathbb{R}^+$, $b > a \rightarrow a^n = o(b^n)$.

Exercise 1-4

Prove or disprove each of the following.

a) $f(n) = O(g(n)) \rightarrow g(n) = O(f(n))$.

b) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.

c) $f(n) = O((f(n))^2)$.

Exercise 1-5

What is the worst-case time complexity of the following algorithm? (Find a tight bound in terms of O or Θ.)

```
Iterator(int n) {
  for(i = 1; i <= n; i++)
    for(j = i; j >= 1; j--)
      for(k = 1; k <= j; k++)
        print(j)
}
```

Exercise 1-6

Using two different methods, prove that $\sum_{i=1}^{n} i^p = \Theta(n^{p+1})$, for $p \geq 1$.

Exercise 1-7

Prove that $\sum_{i=2}^{n} \frac{1}{i^2} = \Theta(1 - \frac{1}{n})$

Exercise 1-8 **Submission**
Due by 13:45, Saturday, September 23rd

Name:

Using two different methods, prove that $\sum_{i=1}^{i=n} 2^i = \Theta(2^n)$.