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**Introduction to Computer Science**, Winter Semester 2017
**Practice Assignment 9A**
Discussion: 16.12.2017 - 21.12.2017

**Exercise 9A-1** To be Discussed in Tutorial

Convert the following numbers from binary to decimal:

a) $(1001001.011)_2$

b) $(101111.0111)_2$

c) $(0.1110100)_2$

**Solution:**

a) $(1001001.011)_2 = 1*2^0 + 1*2^3 + 1*2^6 + 1*2^{-2} + 1*2^{-3} = 73.375$

b) $(101111.0111)_2 = 1*2^0 + 1*2^1 + 1*2^2 + 1*2^3 + 1*2^5 + 1*2^{-2} + 1*2^{-3} + 1*2^{-4} = 47.4375$

c) $(0.1110100)_2 = 1*2^{-1} + 1*2^{-2} + 1*2^{-3} + 1*2^{-5} = 0.90625$

**Exercise 9A-2** To be Discussed in Tutorial

- Convert the following number from decimal to binary: 50.75

  **Solution:**
  $50.75 = 110010.11$

$$\begin{aligned} 0.75 * 2 &= \underline{1}.5 \\ 0.5 * 2 &= \underline{1}.0 \end{aligned}$$

- Show the binary representation of the decimal number 42.74 with approximation of 5-bits after the decimal point.

  **Solution:**
  $42.74 = 101010.10111_2$

$$\begin{aligned} 0.74 * 2 &= \underline{1}.48 \\ 0.48 * 2 &= \underline{0}.96 \\ 0.96 * 2 &= \underline{1}.92 \\ 0.92 * 2 &= \underline{1}.84 \\ 0.84 * 2 &= \underline{1}.68 \end{aligned}$$

**Exercise 9A-3** To be Discussed in Tutorial

Show how the decimal number $-22.246$ is stored in a computer that uses 16 bits to represent real numbers (10 for the mantissa and 6 for the exponent, both including the sign bit). Show your work as indicated below.

a) Show the binary representation of the decimal number $-22.246$.

**Solution:**

$-22.246_{10} = -10110.0011_2$

$$\begin{aligned} 0.246 * 2 &= \underline{0}.492 \\ 0.492 * 2 &= \underline{0}.984 \\ 0.984 * 2 &= \underline{1}.968 \\ 0.968 * 2 &= \underline{1}.936 \end{aligned}$$

b) Show the binary number in normalized scientific notation.

**Solution:**

$-22.246_{10} = -10110.0011_2 = -.101100011 \times 2^5$

c) Show how the binary number will be stored in the 16 bits below.

| | | | |
|---|---|---|---|
| | | | |
| Sign of mantissa 1 bit | Mantissa 9 bits | Sign of exponent 1 bit | Exponent 5 bits |

**Solution:**

| | | | |
|---|---|---|---|
| 1 | 101100011 | 0 | 00101 |
| Sign of mantissa 1 bit | Mantissa 9 bits | Sign of exponent 1 bit | Exponent 5 bits |

**Exercise 9A-4** To be Discussed in Tutorial

Assume that our computer stores decimal numbers using 16 bits — 10 bits for a sign/magnitude mantissa and 6 bits for a sign/magnitude base-2 exponent.

| Sign of mantissa | Mantissa | Sign of exponent | Exponent |
|---|---|---|---|
| 1 bit | 9 bits | 1 bit | 5 bits |

Show the internal representation of the following decimal floating points:

a) 7.5

b) -20.25

c) 0.015625

**Solution:**

a) $7.5 = 111.1 = 0.1111 \times 2^3$

| | | | |
|---|---|---|---|
| 0 | 111100000 | 0 | 00011 |

b) $-20.25 = -10100.01 = 0.1010001 \times 2^5$

| | | | |
|---|---|---|---|
| 1 | 101000100 | 0 | 00101 |

c) $0.015625 = 0.000001 = 0.1 \times 2^{-5}$

| | | | |
|---|---|---|---|
| 0 | 100000000 | 1 | 00101 |

**Exercise 9A-5**

Recall the 16-Bit encoding schema for a normalized scientific binary floating point:

| Sign of mantissa 1 bit | Mantissa 9 bits | Sign of exponent 1 bit | Exponent 5 bits |
|---|---|---|---|

Translate the following numbers into this schema (show your workout):

a) $54272_{10}$

b) $.00011011_2$

c) $10001.011_2$

**Solution:**

a) Translating 54272 into normalized scientific binary floating point. First, translating into binary:

$$\begin{aligned}
54272/2 &= 27136 \quad Rem0\\
27136/2 &= 13568 \quad Rem0\\
13568/2 &= 6784 \quad Rem0\\
6784/2 &= 3392 \quad Rem0\\
3392/2 &= 1696 \quad Rem0\\
1696/2 &= 848 \quad Rem0\\
848/2 &= 424 \quad Rem0\\
424/2 &= 212 \quad Rem0\\
212/2 &= 106 \quad Rem0\\
106/2 &= 53 \quad Rem0\\
53/2 &= 26 \quad Rem1\\
26/2 &= 13 \quad Rem0\\
13/2 &= 6 \quad Rem1\\
6/2 &= 3 \quad Rem0\\
3/2 &= 1 \quad Rem1\\
1/2 &= 0 \quad Rem1
\end{aligned}$$

Thus, $54272 = 1101010000000000_2$. Normalized, that is $.1101010000000000 \times 2^{16}$. Thus, the exponent is16 or $10000_2$. Both mantissa and exponent are positive. The full 16-Bit representation is thus:

| | | | |
|---|---|---|---|
| 0 | 110101000 | 0 | 10000 |

b) $.00011011_2$

| | | | |
|---|---|---|---|
| 0 | 110110000 | 1 | 00011 |

c) $10001.011_2$

| | | | |
|---|---|---|---|
| 0 | 100010110 | 0 | 00101 |

**Exercise 9A-6**

We would like to store the floating-point number $-33.66$ in a computer that uses 16 bits to represent real numbers.

a) The aim now is to find out the number of bits that will be used for the exponent and the mantissa. Assuming that the number of bits to represent the exponent will be the least number of bits needed to represent the exponent for the number $-33.66$. Find

- the total number of bits needed to represent the exponent and

  **Solution:**
  $(33)_{10} = (100001)_2$ thus the exponent should have the value 6 which needs 3 bits to be represented in binary (110). Therefore 4 bits will be needed to represent the exponent including the sign bit.

- the total number of bits to represent the mantissa (assuming that we have in total 16 bits to represent real numbers)

  **Solution:**
  $16 - 4 = 12$ remaining bits to represent the mantissa including the sign bit.

b) Give the largest number in binary that can be represented using the number of bits of the mantissa and the exponent from part a).

**Solution:**

1111111.1111

c) Show the binary representation of the decimal number $-33.66$.

**Solution:**

$-33.66_{10} = -100001.10101_2$

$$
\begin{aligned}
0.66 * 2 &= \underline{1}.32 \\
0.32 * 2 &= \underline{0}.64 \\
0.64 * 2 &= \underline{1}.28 \\
0.28 * 2 &= \underline{0}.56 \\
0.56 * 2 &= \underline{1}.12
\end{aligned}
$$

d) Show the binary number in normalized scientific notation.

**Solution:**

$-33.66_{10} = -0.10000110101 \times 2^6$

e) Show how the binary number will be stored in the 16 bits below.

**Solution:**

| | | | |
|---|---|---|---|
| 1 | 10000110101 | 0 | 110 |
| Sign of mantissa<br>1 bit | Mantissa<br><br>x bits | Sign of exponent<br>1 bit | Exponent<br><br>y bits |

**Exercise 9A-7** To Be Solved in Lab

Given a list of characters represented by their Unicode in decimal, write a Python program that produces the corresponding String by concatenating all characters of the list. Test your algorithm with the following input list:

| | | | |
|---|---|---|---|
| 65203 | 65248 | 65268 | 65250 |

**Solution:**

```
A = eval(input())
n = len(A)
i = 0
B= [""]*n
while(i < n):
  B[i] = chr(A[i])
  i += 1

i=0
while (i < n):
  print(B[i],end="")
  i+=1
```

**Exercise 9A-8**

Given the following RGB representation of a color, represent its corresponding hexadecimal value.

```
R= 255
G= 197
B= 217
```

**Solution:**

Hexadecimal representation: #FFC5D9

**Exercise 9A-9**

Given a list of 1's and 0's representing one row in an image, write a Python program that stores the pixel's color in a list of Strings as black and white. Assume that black pixels are represented as 0's and white pixels as 1's.

**Solution:**

```
A = eval(input())
n = len(A)
i = 0
B= [""]*n
```

```
while(i < n):
  if(A[i]==1):
    B[i] = "W"
  else:
    B[i] = "B"
  i += 1
print(B)
```

**Exercise 9A-10** To Be Solved in Lab

Given a list of 1's and 0's representing one row in an image, write a Python program that inverts the image, i.e. invert white pixels to black and vice versa. Assume that black pixels are represented as 0's and white pixels as 1's.

**Solution:**

```
A = eval(input())
n = len(A)
i = 0
while(i < n):
  A[i] = 1 - A[i]
  i += 1
print(A)
```