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Introduction to Computer Science, Winter Semester 2017
Practice Assignment 9B
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Exercise 9B-1 To be Discussed in Tutorial

What would be the ranges of numbers that can be represented by sign/magnitude, 1's complement, and 2's complement using the following number of bits:

- 3 bits
- 8 bits
- 10 bits

Justify your answer.

Solution:

	Sign/Magnitude	1's Complement	2's Complement
3 bits	$[-3, 3]$	$[-3, 3]$	$[-4, 3]$
8 bits	$[-127, 127]$	$[-127, 127]$	$[-128, 127]$
10 bits	$[-511, 511]$	$[-511, 511]$	$[-512, 511]$

- Sign/Magnitude:
  The formula for calculating the range is $[-2^{n-1} - 1, 2^{n-1} - 1]$. Usually if you have n bits to represent a certain number then you have a range of $[0, 2^n - 1]$ numbers. For instance, if you have 3 bits, then you have $[0, 2^3 - 1]$ which is $[0, 7]$. However, if one bit is reserved for the sign then you have only $n-1$ bits to represent the number itself which yields $[0, 2^{n-1} - 1]$. Taking into consideration negative numbers as well then the range would be $[-2^{n-1} - 1, 2^{n-1} - 1]$. If this is applied on the 3 bits we get $[-3, 3]$ instead of $[-7, 7]$.
  The same idea of the reserved bit holds for the 1's and 2's complement.
- 1's Complement
  The ranges of numbers represented in the 1's complement is the same as that of the sign/magnitude however, numbers represented in 1's complement are better in performing arithmetic operations.
- 2's Complement
  The problem with the 1's complement representations that you have two representations for the the zero: $+0$ and -0. 2's complement representations get rid of representing -0 and thus adding an extra number to the range of its numbers yielding a range of $[-2^{n-1}, 2^{n-1} - 1]$.

Exercise 9B-2 To be Discussed in Tutorial

Write the 8-bit sign magnitude, 1's complement and 2's complement representations for each of these decimal numbers:

a) $+18$

b) +115

c) -49

d) -100

Solution:

Decimal Value	Sign magnitude	1's complement	2's complement
+18	00010010	00010010	00010010
+115	01110011	01110011	01110011
-49	10110001	11001110	11001111
-100	11100100	10011011	10011100

- To determine the sign magnitude representation of -49
  - First determine the binary representation of 49 which is 00110001.
  - Replace the leftmost bit by 1. Thus the sign magnitude representation of -49 is 10110001
- To determine the one's complement representation of -49
  - First determine the binary representation of 49 which is 00110001.
  - Then invert each 1 to 0 and each 0 to 1. Thus the the one's complement representation of -49 is 11001110
- To determine the two's complement representation of -49
  - First determine the binary representation of 49 which is 00110001.
  - Then invert each 1 to 0 and each 0 to 1. We get 11001110.
  - Finally, add 1 to 11001110. Thus the the two's complement representation of -49 is 11001111.

Exercise 9B-3 To be Discussed in Tutorial

Perform the addition of the following binary numbers.

a) $0.011 + 0.0101$

b) $101 + 1.01$

c) $1011 + 1.11$

d) $101.01 + 1011.01$

Solution:

a) $$\begin{array}{cccccc} & 0. & 0 & 1 & 1 & 0 \\ + & 0. & 0 & 1 & 0 & 1 \\ \hline & 0. & 1 & 0 & 1 & 1 \end{array}$$

b) $$\begin{array}{cccccc} & 1 & 0 & 1. & 0 & 0 \\ + & & & 1. & 0 & 1 \\ \hline & 1 & 1 & 0. & 0 & 1 \end{array}$$

c) $$\begin{array}{ccccccc} & 1 & 0 & 1 & 1. & 0 & 0 \\ + & & & & 1. & 1 & 1 \\ \hline & 1 & 1 & 0 & 0. & 1 & 1 \end{array}$$

d) $$\begin{array}{cccccccc} & & 0 & 1 & 0 & 1. & 0 & 1 \\ + & & 1 & 0 & 1 & 1. & 0 & 1 \\ \hline & 1 & 0 & 0 & 0 & 0. & 1 & 0 \end{array}$$

Exercise 9B-4 To be Discussed in Tutorial

Subtract the following 4-bit binary numbers which are represented using the two's complement notation and give the results in the decimal system (base 10).

a) $1011 - 1001$

b) $1100 - 0110$

c) $1010 - 0011$

d) $1011 - 1101$

e) $0111 - 1001$

f) $1100 - 1100$

Solution:

a) $1011 - 1001 = 1011 + 0111 = \underline{1}0010 = 2_{10}$

- 1001 is the binary representation of a negative number since the sign bit is 1.
- To determine the decimal value of 1001, first invert each 1 to 0 and each 0 to 1, we get 0110. Then add 1 to it, we get 0111 that represents 7. Thus 1001 represents -7.
- To do the subtraction $1011 - 1001$ we use the addition $1011 + 0111$. The result is $\underline{1}0010$.
- We discard the final carry: 0010. This binary number represents the decimal number 2

b) $1100 - 0110 = 1100 + 1010 = \underline{1}0110 = 6_{10}$

c) $1010 - 0011 = 1010 + 1101 = \underline{1}0111 = 7_{10}$

d) $1011 - 1101 = 1011 + 0011 = 1110 == -0010 = -2_{10}$

e) $0111 - 1001 = 0111 + 0111 = 1110 = -0010 = -2_{10}$

f) $1100 - 1100 = 1100 + 0100 = \underline{1}0000 = 0_{10}$

Exercise 9B-5 To be Discussed in Tutorial

Assume that our computer stores decimal numbers using 8 bits. Perform the following subtractions using 2's complement notation:

a) $26 - 13$

b) $29 - 36$

c) $18 - 19$

Solution:

a) $26 - 13 = 26 + (-13)$.

- Two's complement of 26 is 00011010
- Two's complement of 13 is 00001101
- Two's complement of -13 is 11110011
- $00011010 + 11110011 = \underline{1}00001101$

- Discard the final carry: 00001101
- The decimal value of 00001101 is 13

$$\begin{aligned} 26 - 13 &= 00011010 - 00001101 \\ &= 00011010 + 11110011 \\ &= \underline{1}00001101 \\ &= 13_{10} \end{aligned}$$

b) $29 - 36 = 29 + (-36)$

$$\begin{aligned} 29 - 36 &= 00011101 - 00100100 \\ &= 00011101 + 11011100 \\ &= 11111001 \\ &= -00000111 \\ &= -7_{10} \end{aligned}$$

c) $18 - 19 = 18 + (-19)$

$$\begin{aligned} 18 - 19 &= 00010010 - 00010011 \\ &= 00010010 + 11101101 \\ &= 11111111 \\ &= -00000001 \\ &= -1_{10} \end{aligned}$$

Exercise 9B-6 To be Discussed in Tutorial

Assume that our computer stores decimal numbers using 5 bits. Perform the following operation using 2's complement notation:

$-13 - 12$

Solution:

$-13 - 12 = (-13) + (-12)$

- Two's complement of 13 is 01101
- Two's complement of -13 is 10011
- Two's complement of 12 is 01100
- Two's complement of -12 is 10100
- $10011 + 10100 = \underline{1}00111$
- Discard the final carry: 00111
- The decimal value of 00111 is 7

$$\begin{aligned} -13 + -12 &= 10011 + 10100 \\ &= \underline{1}00111 \\ &= 7_{10} \end{aligned}$$