German University in Cairo
Computer Science and Engineering Department
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Introduction to Computer Science, Winter Term 2006-2007

Final Exam

Bar Code

Instructions: Read carefully before proceeding.

1) Duration of the exam: 3 hours (180 minutes).
2) (Non-programmable) Calculators are allowed.
3) No books or other aids are permitted for this test.
4) This exam booklet contains 14 pages, including this one. Three extra sheets of scratch paper are attached and have to be kept attached. **Note that if one or more pages are missing, you will lose their points. Thus, you must check that your exam booklet is complete**.
5) Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem or on the three extra sheets and make an arrow indicating that. **Scratch sheets will not be graded unless an arrow on the problem page indicates that the solution extends to the scratch sheets**.
6) When you are told that time is up, stop working on the test.

Good Luck!

Don't write anything below ;-)

Exercise	1	2	3	4	5	6	7	8	9	10	$\sum$
Possible Marks	5	6	6	8	6	8	8	6	13	8	74
Final Marks											

Exercise 1 (5 Marks)

Mark the correct answer in the following (**only one choice**):

1) Main memory is considered volatile because
   - [ ] it may change at any time without warning.
   - [x] the contents of memory are lost when the computer is turned off.
   - [ ] the contents of memory are the same each time the computer is turned on.
   - [ ] the memory may be easily removed from the computer.

2) A bus is
   - [ ] part of the computer that decides if a value should be stored as an integer or floating point.
   - [x] a group of parallel wires that carry control signals and data between the computer's components.
   - [ ] a logical operation that can be performed by a computer.
   - [ ] a series of tests that is performed on all of the computer's components during machine startup.

3) Each cell of memory is numbered and that number is referred to as the cell's
   - [ ] block.
   - [ ] identity.
   - [x] address.
   - [ ] size.

4) Main memory is called RAM because
   - [ ] it is volatile, like a ram's temper.
   - [ ] the computer starts at address 0 and reads every byte until it reaches the correct address.
   - [ ] it can Read All Memory.
   - [x] the memory is accessible randomly.

5) Assume that the memory size of a computer is 2^{26} bits and the address width is 6 bits. How many bits does a cell have?
   - [x] 2^{20}
   - [ ] 2^{19}
   - [ ] 20
   - [ ] 19
   - [ ] None of the above

Exercise 2 (6 Marks)

a) Convert the binary number 101101_2 to a decimal number (base 10). Show your workout.

Solution:

$2^5 + 2^3 + 2^2 + 2^1 = 32 + 8 + 4 + 1 = 45_{10}$

b) Convert the decimal number 61_{10} to a number in base 5. Show your workout.

Solution:

Division	Quotient	Remainder
61/5	12	1
12/5	2	2
2/5	0	2

$54_{10} = 221_5$

c) Convert the octal number 456_8 to a binary number. Show your workout.

Solution:

4	5	6
100	101	110

Thus $456_8 = 100101110_2$

Exercise 3 (6 Marks)

Show how the decimal number -25.333 is stored in a computer that uses 16 bits to represent real numbers (10 for the mantissa and 6 for the exponent, both including the sign bit). Show your work as indicated below.

a) Show the binary representation of the decimal number -25.333.

Solution:

$-25.333_{10} = -11001.0101_2$

$$\begin{aligned} 0.333 * 2 &= \underline{0}.666 \\ 0.666 * 2 &= \underline{1}.332 \\ 0.332 * 2 &= \underline{0}.664 \\ 0.664 * 2 &= \underline{1}.328 \end{aligned}$$

b) Show the binary number in normalized scientific notation.

Solution:

$-25.333_{10} = -11001.0101_2 = -.110010101 \times 2^5$

c) Show how the binary number will be stored in the 16 bits below.

Solution:

1	110010101	0	00101
Sign of mantissa 1 bit	Mantissa 9 bits	Sign of exponent 1 bit	Exponent 5 bits

Exercise 4 (8 Marks)

a) Assume that our computer stores decimal numbers using 4 bits. Determine the range (in decimal) of binary numbers and represent the range in binary.

- Unsigned binary numbers

  Solution:

  - Range for unsigned numbers: $[0, 2^4 - 1] = [0, 15]$
  - $0_{10} = 0000_2$
  - $15_{10} = 1111_2$

- Signed binary numbers in sign-magnitude notation

  Solution:

  - Range for signed numbers in sign-magnitude notation: $[-(2^{4-1} - 1), 2^{4-1} - 1] = [-7, 7]$
  - $7_{10} = 0111_2$
  - $-7_{10} = 1111_2$

- Signed binary numbers in one's complement notation

  Solution:

  - Range for signed numbers in one's complement notation: $[-(2^{4-1} - 1), 2^{4-1} - 1] = [-7, 7]$
  - $7_{10} = 0111_2$
  - $-7_{10} = 1000_2$

- Signed binary numbers in two's complement notation

  Solution:

  - Range for signed numbers in two's complement notation: $[-2^{4-1}, 2^{4-1} - 1] = [-8, 7]$
  - $7_{10} = 0111_2$
  - $-8_{10} = 1000_2$

Exercise 5 (6 Marks)

Assume that our computer stores decimal numbers using 5 bits. Perform the subtraction

$$(-14)_{10} - (9)_{10}$$

using 2's complement notation. Give the result of the subtraction in decimal. Show your workout, i.e. all steps performed.

Solution:

- Convert 14 and 9 to binary:

  $14_{10} = 01110_2$
  $9_{10} = 01001_2$

- Two's complement representation of -14

  $-14_{10} = 10010$

- Two's complement representation of -9

  $-9_{10} = 10111$

- Perform the addition $(-14)_{10} - (9)_{10}$ in binary:

  $10010 + 10111 = 101001$

- Remove the overflow:

  $10010 + 10111 = 01001$

- The binary number 01001 represents the positive decimal value 9.

- $(-14)_{10} - (9)_{10} = 9_{10}$

Exercise 6 (8 Marks)

Given the following the following truth table, where **A**, **B** and **C** are the input variables and **X** is the output variable.

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

a) Use the sum-of-products algorithm to find the Boolean expression that describes the output of the truth table.

Solution:

$$X = A'B'C' + A'BC' + AB'C' + ABC$$

b) What is the functionality of the circuit?

Solution:

The circuit will perform the following logical operation:

$$A * B = C$$

c) Draw the Boolean circuit. **Note** that each gate can have only two inputs.

Exercise 7 (8 Marks)

A circuit should be designed to perform the moduls operation of two numbers consisting of two bits each. Assume that for any number N, $N\%0 = 3$.

a) How many input and output variables are needed?

Solution:

We need 4 input variables and 2 output variables

b) Construct the truth table for this circuit

Solution:

A	B	C	D	O1	O2
0	0	0	0	1	1
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	0	1	0	0
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	0	0

c) Using the sum-of-products method, find the Boolean expressions that correspond to the constructed truth table.

Solution:

$$O1 = A'B'C'D' + A'BC'D' + AB'C'D' + AB'CD + ABC'D'$$

$$O2 = A'B'C'D' + A'BC'D' + A'BCD' + A'BCD + AB'C'D' + ABC'D' + ABCD'$$

Exercise 8 (6 Marks)

Simplify the Boolean expression using the Boolean algebra. **Note that both simplified expressions consist of one gate each.**

Please mention the applied rules.

$x + 0 = x$	$x * 1 = x$	
$x + 1 = 1$	$x * 0 = 0$	
$x + x = x$	$x * x = x$	
$x + x' = 1$	$x * x' = 0$	
$(x')' = x$		
$x + y = y + x$	$xy = yx$	Commutativity
$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$	Associativity
$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$	Distributivity
$(x + y)' = x'y'$	$(xy)' = x' + y'$	DeMorgan's Law

a) $(A + C) * (AD + AD') + AC + C$

Solution:

$$\begin{aligned}
& (A + C) * (AD + AD') + AC + C \\
&= (A + C) * (A * (D + D')) + AC + C \\
&= (A + C) * (A * 1) + AC + C \\
&= (A + C) * A + AC + C \\
&= (A + C) * A + AC + C \\
&= (A + C) * A + CA + C \\
&= (A + C) * A + CA + C * 1 \\
&= (A + C) * A + C * (A + 1) \\
&= (A + C) * A + C * 1 \\
&= (A + C) * A + C \\
&= A * (A + C) + C \\
&= AA + AC + C \\
&= A + AC + C * 1 \\
&= A + C * A + C * 1 \\
&= A + C * (A + 1) \\
&= A + C * 1 \\
&= A + C
\end{aligned}$$

b) $A' * (A + B) + (B + AA) * (A + B')$

Solution:

$$\begin{aligned}
& A' * (A + B) + (B + AA) * (A + B') \\
&= A' * (A + B) + (B + A) * (A + B') \\
&= A' * A + A'B + (B + A) * (A + B') \\
&= A * A' + A'B + (B + A) * (A + B') \\
&= 0 + A'B + (B + A) * (A + B') \\
&= A'B + (B + A) * (A + B') \\
&= A'B + (B + A) * A + (B + A) * B' \\
&= A'B + BA + AA + BB' + AB'
\end{aligned}$$

$$
\begin{aligned}
&= A'B + BA + AA + 0 + AB' \\
&= A'B + BA + AA + AB' \\
= A'B + BA + A + AB' &= A'B + AB + A * 1 + AB' \\
&= A'B + A(B + 1) + AB' \\
&= A'B + A * 1 + AB' \\
&= A'B + A(1 + B') \\
&= A'B + A * (B' + 1) \\
&= A'B + A * 1 \\
&= A'B + A \\
&= (A + A') * (A + B) \\
&= 1 * (A + B) \\
&= (A + B) * 1 \\
&= A + B
\end{aligned}
$$

Exercise 9 (8+5 (Bonus) Marks)

Given a number in binary, write an algorithm to divide the number by 4. The input of the algorithm is a list (array) of 0'a and 1's amd the result should be stored in a list (or array) too.

For example, for the number

```
1 0 0 0
```

your algorithm should store the result into an array

```
1 0
```

For a solution that will use only one loop 5 bonus marks will be given.

Solution:

```
get n
get A1, A2, ..., An
set i to n
set j to n - 2
while (i > 2)
{
   set Bj to Ai-2
   set i to i - 1
   set j to j - 1
}
```

Exercise 10 (8 Marks)

Given the following algorithm

```
get x
get n
set i to n - 1
while (i >= 0) do
   {
      if (x-2^i) < 0
      then
         set Bi to 0
      else
         set Bi to 1;
         set x to x-2^i
      endif
      set i to i-1
   }
set i to i + 1
while(i < n)
    {
       print Bi
       set i to i + 1
    }
```

Please note that `2^i` corresponds to the power operation 2^i.

a) What is the output of the algorithm for `x = 21` and `n = 5`. Show your workout.

Solution:

The algorithm will generate the following list

```
B4 B3 B2 B1 B0
1  0  1  0  1
```

b) What is the output of the algorithm for any numbers `x` and `n`?

Solution:

The algorithm calculates the binary representation of `x` and represent it in `n` bits.

c) State a condition about `n` in order that the algorithm will produce always correct answers.

Solution:

$x \leq 2^n$

d) Determine the order of magnitude of the algorithm. Justify your answer.

Solution:

The while loop will be exexuted n times thus the complexity is $O(n)$.

Extra Page

Extra Page

Extra Page