

Exercise 1

(10 Marks)

Write an algorithm that plays a guessing game with numbers. The goal is for the user to guess a randomly selected number between 1 and 100 as quickly as possible. The algorithm will inform the user whether the hidden number is higher or lower than the current guess. You can assume that the user will always enter integers, but you must verify that the integers entered are between 1 and 100.

The following is a sample run of the program:

```
I'm thinking of a number between 1 and 100.  
Enter guess: 50  
Number is higher than 50  
Enter guess: 588  
Invalid, try again: 75  
Number is lower than 75  
Enter guess: 62  
Number is lower than 62  
Enter guess: 56  
You guessed it in 4 tries!
```

Complete the following algorithm and make sure that the algorithm **must** return the output according to the above sample.

```
set hiddenNumber to 56
```

Exercise 2

(15 Marks)

In this question you are asked to create a sorting algorithm for lists of integers with a very particular property.

Suppose we know that the list to be sorted contains at most two distinct integer values, each repeated many times. For example the list might be

[12, 12, 933, 933, 12, 933, 12]

This list contains four 12's and three 933's. The sorted form of this list is

[12, 12, 12, 12, 933, 933, 933]

Write an algorithm that will take a list of this form and returns a list containing the same set of values in sorted order.

The algorithm should run in linear time; $O(n)$ where n is the number of elements in the list.

Exercise 3

(5+3+5+1=14 Marks)

Given the following algorithm

```
get n
get A1, ..., An
set k to 1
set i to 2
while (i <= n )
{ if (Ak <> Ai)
  then
    print Ak
  endif
  set i to i + 1
  set k to k + 1
}
set i to i - 2
if (Ak <> Ai)
then print Ak
endif
```

a) What is the output of the algorithm for the following list:

1 1 1 2 3 4 4 5 6

Use a tracing table to trace the first while loop.

Output of the algorithm:b) What is the output of the algorithm for any sorted list of the form A_1, \dots, A_n ?

- c) Find the total number of executed operations in worst case. State explicitly the worst case. Show your workout.

```
get n
get A1, ..., An
set k to 1 -----> 1 operation executed once
set i to 2 ----->
while (i <= n) ----->
{ if (Ak <> Ai)----->
  then
    print Ak ----->
  endif
  set i to i + 1 ----->
  set k to k + 1 ----->
}
set i to i - 2 ----->
if (Ak <> Ai) ----->
then print Ak ----->
endif
```

Total number of operations:

- d) Determine the order of magnitude of the algorithm.

Exercise 4

(2+2+2=6 Marks)

- a) Convert the binary number 101101_2 which is in two's complement to a decimal number (base 10). Show your workout.

- b) Convert the decimal number 236_{10} to a number in base 8. Show your workout.

- c) Convert the octal number 65432_8 to a hexadecimal number. Show your workout.

Exercise 5

(3+2+3=8 Marks)

We would like to store the floating-point number -19.55 in a computer that uses 16 bits to represent real numbers. (10 for the mantissa and 6 for the exponent, both including the sign bit). Show your work as indicated below.

a) Show the binary representation of the decimal number -19.55 .

b) Show the binary number in normalized scientific notation.

c) Show how the binary number will be stored in the 16 bits below.

Sign of mantissa 1 bit	Mantissa 9 bits	Sign of exponent 1 bit	Exponent 5 bits

Exercise 6

(6 Marks)

Assume that our computer stores decimal numbers using 6 bits. Perform the subtraction

$$(-19)_{10} - (18)_{10}$$

using 2's complement notation. Give the result of the subtraction in decimal. Show your workout, i.e. all steps performed.

Exercise 7

(5+4+5=14 Marks)

Given the following truth table, where **A**, **B** are the input variables and **X**, **Y**, and **Z** are the output variables.

A	B	X	Y	Z
0	0	0	0	0
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

- a) Use the sum-of-products algorithm to find the Boolean expressions that describe the output of the truth table.

- b) What is the functionality of the circuit?

- c) Draw the Boolean circuit. **Note** that each gate can have only two inputs.

Exercise 8

(6+4=10 Marks)

- a) Using fulladder circuits, design a circuit that adds two 4-bit numbers A and B .
- b) Design a circuit that will subtract two 4-bit numbers A and B from each other ($A - B$) where the numbers are in 1's complement. You are only allowed to use the circuit you designed in part a) and negation gates.

Exercise 9

(7 Marks)

Given the following Boolean expression

$$A'(A + B) + (B + AA)(A + B')$$

Simplify the Boolean expressions using the Boolean algebra. Please mention the applied rules.

$x + 0 = x$	$x * 1 = x$	
$x + 1 = 1$	$x * 0 = 0$	
$x + x = x$	$x * x = x$	
$x + x' = 1$	$x * x' = 0$	
$(x')' = x$		
$x + y = y + x$	$xy = yx$	Commutativity
$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$	Associativity
$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$	Distributivity
$(x + y)' = x'y'$	$(xy)' = x' + y'$	DeMorgan's Law

Hint: The circuit of the simplified expression consists of only one gate.

Extra Page

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