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Introduction to Computer Science, Winter Semester 2018
Practice Assignment 10

Discussion: 29.12.2018 - 03.01.2018

Exercise 10-1

Given the following truth table, where **A**, **B** and **C** are the input variables and **X** is the output variable

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

a) Use the sum-of-products algorithm to find the Boolean expression that describes the output of the truth table.

Solution:

X = AB'C' + AB'C + ABC' + ABC

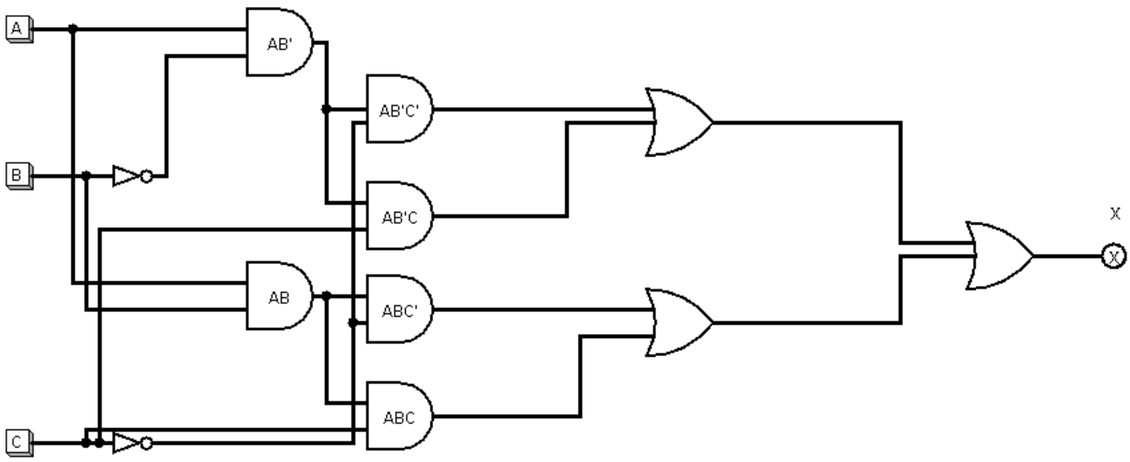
b) What is the functionality of the circuit? you should express it in form of an arithmetic operation.

Solution:

Assuming that input variables correspond to a decimal number N from 0 to 7, the circuit will perform an intger division X = N / 4 it could also be described as a logical operation N ≥4

c) Draw the Boolean circuit.
Note: each gate can have only two inputs.

Solution:



d) Simplify the Boolean expression using the Boolean algebra. Specify the applied rules.

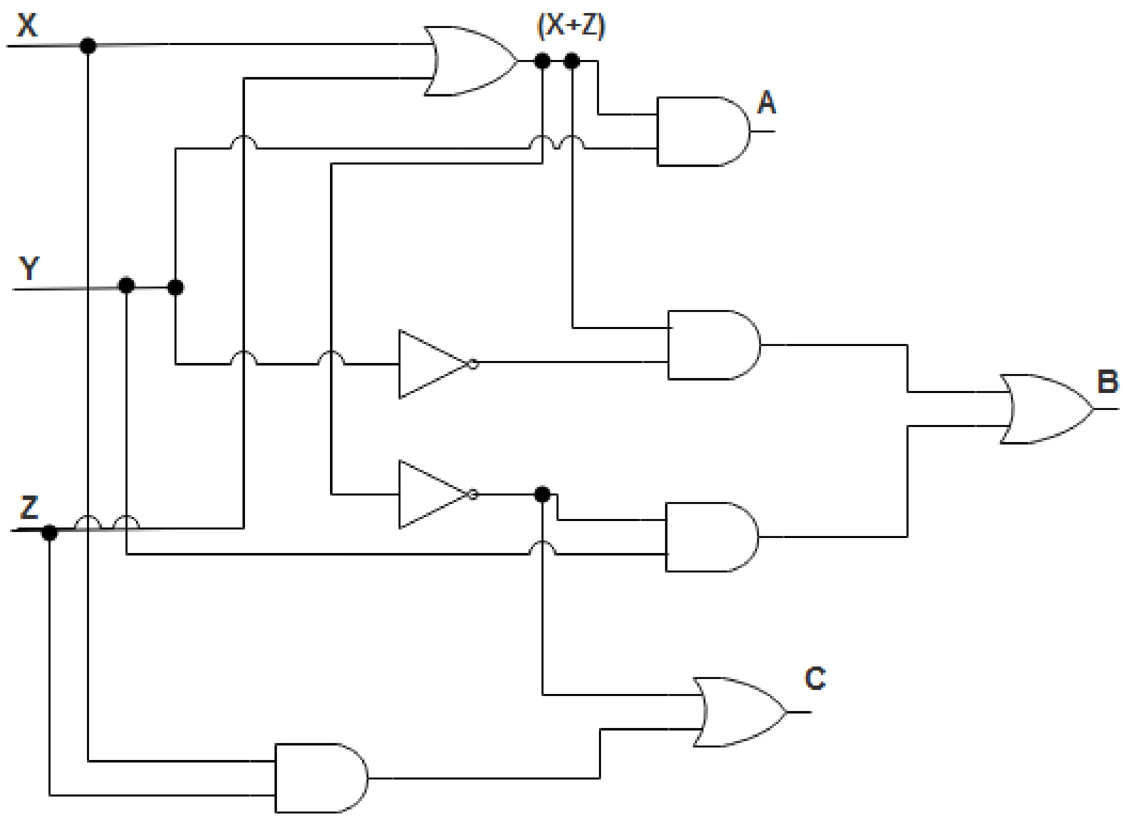
$x + 0 = x$	$x * 1 = x$	
$x + 1 = 1$	$x * 0 = 0$	
$x + x = x$	$x * x = x$	
$x + x' = 1$	$x * x' = 0$	
$(x')' = x$		
$x + y = y + x$	$xy = yx$	Commutativity
$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$	Associativity
$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$	Distributivity
$(x + y)' = x'y'$	$(xy)' = x' + y'$	DeMorgan's Law

Solution:

AB'C' + AB'C + ABC' + ABC	
AB'C' + AB'C + AB (C'+C)	(Distributivity)
AB'C' + AB'C + AB * 1	(x+x' = 1)
AB'C' + AB'C + AB	(x * 1 = x)
AB' (C' + C) + AB	(Distributivity)
AB' * 1 + AB ()	(x+x' = 1)
AB' + AB	(x * 1 = x)
A (B'+B)	(Distributivity)
A * 1	(x+x' = 1)
A	

Exercise 10-2

Given the following circuit



a) Extract the three Boolean expressions for A, B and C from the circuit.

Solution:

$$\text{A} = \text{y(x + z)}$$
$$\text{B} = \text{y(x + z)' + y'(x + z)}$$
$$\text{C} = \text{xz + (x + z)'}$$

b) Draw a truth table for the circuit.

Solution:

X	Y	Z	A	B	C
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1

c) Use the sum-of-products algorithm to determine the Boolean expressions that correspond to the truth table.

Solution:

$$\text{A} = \text{x'yz + xyz' + xyz}$$
$$\text{B} = \text{x'y'z + x'yz' + xy'z'+ xy'z}$$
$$\text{C} = \text{x'y'z' + x'yz' + xy'z + xyz}$$

d) Simplify the Boolean expressions received from part (c) to reach the expressions in part (a). Use the axioms of the Boolean Algebra. Please mention the applied rules.

$x + 0 = x$	$x * 1 = x$	
$x + 1 = 1$	$x * 0 = 0$	
$x + x = x$	$x * x = x$	
$x + x' = 1$	$x * x' = 0$	
$(x')' = x$		
$x + y = y + x$	$xy = yx$	Commutativity
$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$	Associativity
$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$	Distributivity
$(x + y)' = x'y'$	$(xy)' = x' + y'$	DeMorgan's Law

Solution:

- *A = x'yz + xyz'+ xyz*
  = x'yz + xy(z'+ z) (Distributivity)
  = x'yz + xy (x + x'= 1)
  = yx'z + yx (Commutativity)
  = y(x'z + x) (Distributivity)
  = y((x' + x)(z + x)) (Distributivity)
  = y(z + x) (x + x' = 1)

- *B = x'y'z + x'yz'+ xy'z' + xy'z*
  = x'y'z + x'yz' + xy'(z' + z) (Distributivity)
  = x'y'z + x'yz' + xy' (x + x' = 1)
  = x'y'z + xy' + x'yz' (Commutativity)
  = y'x'z + y'x + x'yz' (Commutativity)
  = y'(x'z + x) + x'yz' (Distributivity)

= y'((x' + x)(z + x)) + x'yz' (Distributivity)
= y' (z + x) + x yz' (x + x'= 1)
= y'(z + x) + yx'z' (Commutativity)
= y (z + x) + y(x + z)' (DeMorganâs Law)

- *C = x'y'z' + x'yz' + xy'z + xyz*
  = x'z'y' + x'z'y + xzy' + xzy (Commutativity)
  = x'z'(y'+ y) + xz(y' + y) (Distributivity)
  = x'z' + xz (x + x'= 1)
  = (x + z)' + xz (DeMorganâs Law)

Exercise 10-3

Given the following truth table, where P, X, and Y are the input variables and S and C are the output variables:

P	X	Y	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a) Use the sum-of-products-algorithm to determine the boolean expressions that correspond to the truth table.

Solution:

a) Using the sum-of-products-algorithm

$$S = P'X'Y + P'XY' + PX'Y' + PXY$$
$$C = P'XY + PX'Y + PXY' + PXY$$

Exercise 10-4

Using truth tables, show that:

$X'Y + Y'Z + XZ' = XY' + YZ' + X'Z$

Solution:

- Truth table for $X'Y + Y'Z + XZ'$

X	Y	Z	X'Y	Y'Z	XZ'	X'Y + Y'Z + XZ'
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	1	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	1	0	1	0	1
1	1	0	0	0	1	1
1	1	1	0	0	0	0

- Truth table for $XY' + YZ' + X'Z$

X	Y	Z	XY'	YZ'	X'Z	XY' + YZ' + X'Z
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	1
1	0	1	1	0	0	1
1	1	0	0	1	0	1
1	1	1	0	0	0	0

Exercise 10-5

A circuit should be designed to perform the operation $(A - 1)$ where A represents a number in sign/magnitude notation consisting of 2 bits.

a) How many output variables are needed? Justify your answer.

Solution:

3 output variables are needed. 2 bits in sign magnitude can represnet a range of $[-1, 1]$. Thus the output of calculating $-1 - 1$, which is equal to -2, needs 3 bits to represent it in sign/magnitude notation (110).

b) Construct the truth table for this circuit.

Solution:

A1	A2	O1	O2	O3
0	0	1	0	1
0	1	0	0	0
1	0	1	0	1
1	1	1	1	0

c) Use the sum-of-products algorithm to find the boolean expressions that corresponds to the truth table.

Solution:

$$\begin{aligned} O1 = &\ A1'A2' \quad + A1A2' + A1A2 \\ O2 = &\ A1A2 \\ O3 = &\ A1'A2' \quad + A1A2' \end{aligned}$$

Exercise 10-6

A circuit should be designed to perform the modulus operation of two numbers consisting of two bits each.

Assume that for any number N, $N\%0 = 3$.

a) How many input and output variables are needed?

Solution:

Four input variables and two output variables are needed.

b) Construct the truth table for this circuit

Solution:

A	B	C	D	O1	O2
0	0	0	0	1	1
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	0	1	0	0
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	0	0

c) Using the sum-of-products method, find the Boolean expressions that correspond to the constructed truth table.

Solution:

$$O1 = A'B'C'D' + A'BC'D' + AB'C'D' + AB'CD + ABC'D'$$

$$O2 = A'B'C'D' + A'BC'D' + A'BCD' + A'BCD + AB'C'D' + ABC'D' + ABCD'$$

Exercise 10-7

Simplify the Boolean expressions to a minimum number of literals using the Boolean algebra. Please mention the applied rules.

- $ABC + ABC' + A'B$

  Solution:

$$\begin{aligned}
&ABC + ABC' + A'B \\
&= AB(C + C') + A'B && \textbf{(Distributivity)} \\
&= AB * 1 + A'B && [x + x' = 1] \\
&= AB + A'B && [x * 1 = x] \\
&= BA + BA' && \textbf{(Commutativity)} \\
&= B(A + A') && \textbf{(Distributivity)} \\
&= B * 1 && [x + x' = 1] \\
&= B && [x * 1 = x]
\end{aligned}$$

- $(A + B)'(A' + B')$

  Solution:

$$
\begin{aligned}
&(A+B)'(A'+B') \\
&= (A'B')(A'+B') && [(x+y)' = x'y'] \\
&= A'B'A' + A'B'B' && \textbf{(Distributivity)} \\
&= A'A'B' + A'B'B' && \textbf{(Commutativity)} \\
&= A'B' + A'B' && [x * x = x] \\
&= A'B' && [x + x = x] \\
&= (A+B)' && \textbf{(DeMorgan's Law)}
\end{aligned}
$$

- $(A + B' + AB')(AB + A'C + BC)$

  Solution:

$$
\begin{aligned}
&(A + B' + AB')(AB + A'C + BC) \\
&= (A + B'(1+A))(AB + BC + A'C) && \textbf{(Distributivity)} \\
&= (A + B'(A+1))(AB + BC + A'C) && \textbf{(Commutativity)} \\
&= (A + B' * 1)(AB + BC + A'C) && [(x+1) = 1] \\
&= (A + B')(AB + BC + A'C) && [(x*1) = x] \\
&= (A+B') * AB + (A+B') * BC + (A+B') * A'C && \textbf{(Distributivity)} \\
&= AB * (A+B') + BC * (A+B') + A'C * (A+B') && \textbf{(Commutativity)} \\
&= ABA + ABB' + BCA + BCB' + A'CA + A'CB' && \textbf{(Distributivity)} \\
&= AAB + ABB' + BCA + BB'C + AA'C + A'CB' && \textbf{(Commutativity)} \\
&= AB + ABB' + BCA + BB'C + AA'C + A'CB' && [(x*x) = x] \\
&= AB + 0 + BCA + 0 + 0 + A'CB' && [(x*x') = 0] \\
&= AB + BCA + A'CB' && [(x+0) = x] \\
&= AB + ABC + A'B'C && \textbf{(Commutativity)} \\
&= AB(1+C) + A'B'C && \textbf{(Distributivity)} \\
&= AB(C+1) + A'B'C && \textbf{(Commutativity)} \\
&= AB * 1 + A'B'C && [(x+1) = 1] \\
&= AB + A'B'C && [(x*1) = x]
\end{aligned}
$$

- $P'XY + PX'Y + PXY' + PXY$

  Solution:

$$
\begin{aligned}
&P'XY + PX'Y + PXY' + PXY \\
&= PXY + P'XY + PX'Y + PXY' && \textbf{(Commutativity)} \\
&= XYP + XYP' + PX'Y + PXY' && \textbf{(Commutativity)} \\
&= XY(P+P') + PX'Y + PXY' && \textbf{(Distributivity)} \\
&= XY * 1 + PX'Y + PXY' && [(x+x') = 1] \\
&= XY + PX'Y + PXY' && [(x*1) = x] \\
&= XY + P(X'Y + XY') && \textbf{(Distributivity)}
\end{aligned}
$$

- $(AB)'(A+B)$

  Solution:

$$
\begin{aligned}
&(AB)'(A+B) \\
&= (A'+B')(A+B) && [(xy)' = x'+y'] \\
&= (A'+B')A + (A'+B')B && \textbf{(Distributivity)} \\
&= A(A'+B') + B(A'+B') && \textbf{(Commutativity)} \\
&= AA' + AB' + BA' + BB' && \textbf{(Commutativity)} \\
&= 0 + AB' + BA' + 0 && [(x * x' = 0)] \\
&= AB' + 0 + BA' + 0 && \textbf{(Commutativity)} \\
&= AB' + BA' && [(x + 0 = x)]
\end{aligned}
$$

To construct a circuit for $(AB)'(A+B)$, we will need 4 gates. To construct a circuit for $AB' + BA'$, we need 5 gates. Thus $(AB)'(A+B)$ is simpler than $AB' + BA'$.

- $B + A'C + AB'$

  Solution:

$$
\begin{aligned}
&B + A'C + AB' \\
&= B + AB' + A'C && \textbf{(Commutativity)} \\
&= B + B'A + A'C && \textbf{(Commutativity)} \\
&= (B+B')(B+A) + A'C && [(x+yz) = (x+y)(x+z)] \\
&= 1 * (B+A) + A'C && [(x + x' = 1)] \\
&= (B+A) * 1 + A'C && \textbf{(Commutativity)} \\
&= (B+A) + A'C && [(x * 1 = x)] \\
&= A + A'C + B && \textbf{(Commutativity)} \\
&= (A+A')(A+C) + B && [(x+yz) = (x+y)(x+z)] \\
&= 1 * (A+C) + B && [(x + x' = 1)] \\
&= (A+C) * 1 + B && \textbf{(Commutativity)} \\
&= A + C + B && [(x * 1 = x)]
\end{aligned}
$$

- $AB + A'C + BC$

  Solution:

$$
\begin{aligned}
&AB + A'C + BC \\
&= AB + A'C + BC * 1 && [(x * 1 = x)] \\
&= AB + A'C + BC(A + A') && [(x + x' = 1)] \\
&= AB + A'C + BCA + BCA' && \textbf{(Distributivity)} \\
&= AB + A'C + ABC + A'CB && \textbf{(Commutativity)} \\
&= AB + ABC + A'C + A'CB && \textbf{(Commutativity)} \\
&= AB(1+C) + A'C(1+B) && \textbf{(Distributivity)} \\
&= AB(C+1) + A'C(B+1) && \textbf{(Commutativity)} \\
&= AB * 1 + A'C * 1 && [(x + 1 = 1)] \\
&= AB + A'C
\end{aligned}
$$

Exercise 10-8

Given the following Boolean expression, simplify it to a minimum number of literals using the Boolean algebra. Please mention the applied rules.

$$((A + B)(B' + C' + D')) + B'C'(A + B' + C) + A'C + D$$

Hint: The circuit of the simplified expression consists of zero gates.

Solution:

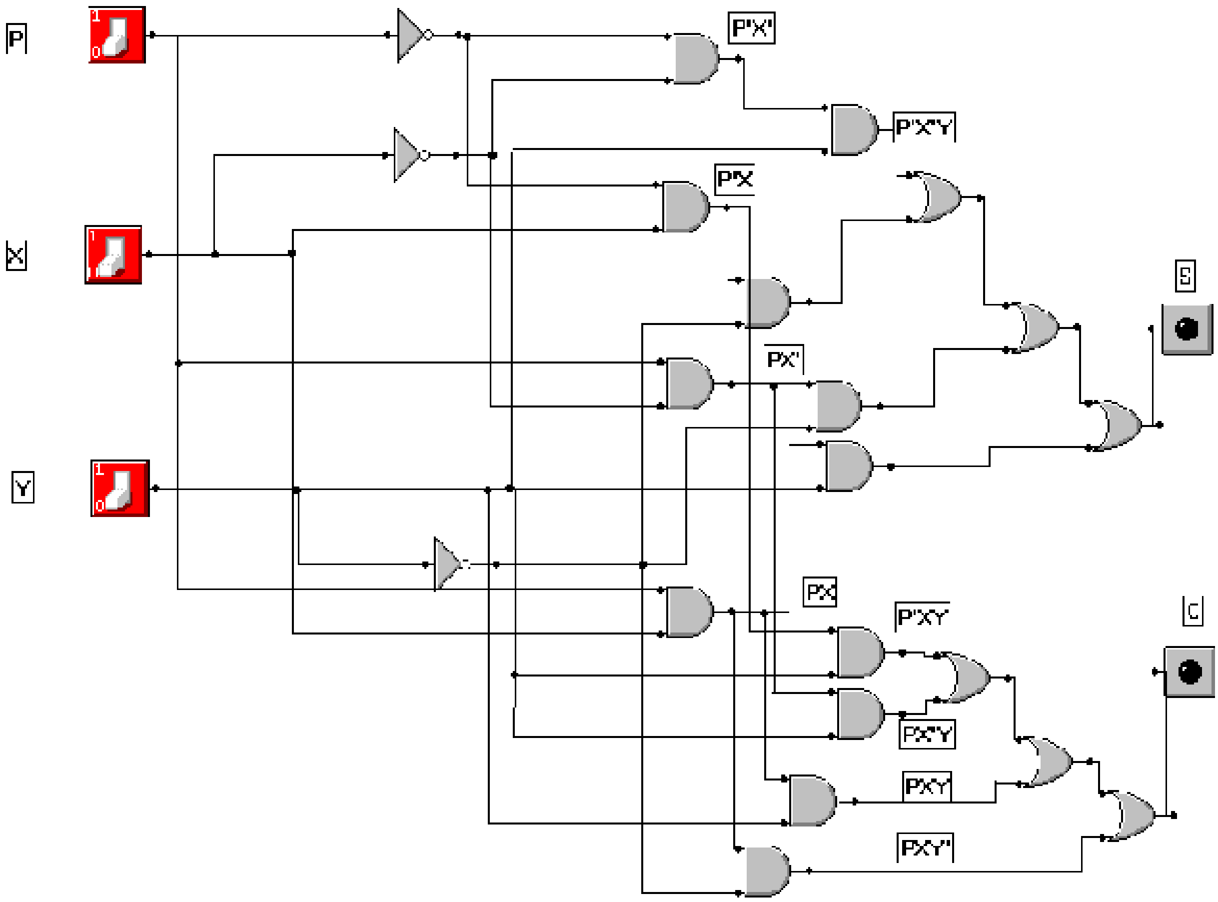
$$\begin{aligned}
&= AB' + AC' + AD' + BB' + BC' + BD' + AB'C' + B'B'C' + B'C'C + A'C + D && (Distributivity)\\
&= AB' + AC' + BB' + BC' + BD' + AB'C' + B'C' + B'C'C + A'C + D + AD' && (Associativity)\\
&= AB' + AC' + BB' + BC' + BD' + AB'C' + B'C' + B'C'C + A'C + (D + A)(D + D') && (Distributivity)\\
&= AB' + AC' + BB' + BC' + BD' + AB'C' + B'C' + B'C'C + A'C + (D + A)(1) && (x + x' = 1)\\
&= AB' + AC' + BB' + BC' + BD' + AB'C' + B'C' + B'C'C + A'C + D + A && (x * 1 = x)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + A'C + D + A + BC' + B'C' && (Associativity)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + A'C + D + A + C'(B + B') && (Distributivity)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + A'C + D + A + C'(1) && (x + x' = 1)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + A'C + D + A + C' && (x * 1 = x)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + A + C' + A'C && (Associativity)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + A + (C' + A')(C' + C) && (Distributivity)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + A + (C' + A')(1) && (x + x' = 1)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + A + C' + A' && (x * 1 = x)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + C' + (A + A') && (Associativity)\\
&= AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + C' + 1 && (x + x' = 1)\\
&= (AB' + AC' + BB' + BD' + AB'C' + B'C'C + D + C') + 1 && (x + 1 = 1)\\
&= 1
\end{aligned}$$

Exercise 10-9

Use AND, OR and NOT gates to implement the circuits represented by the following two expressions:

$$\begin{aligned}
S &= P'X'Y + P'XY' + PX'Y' + PXY\\
C &= P'XY + PX'Y + PXY' + PXY
\end{aligned}$$

Solution:

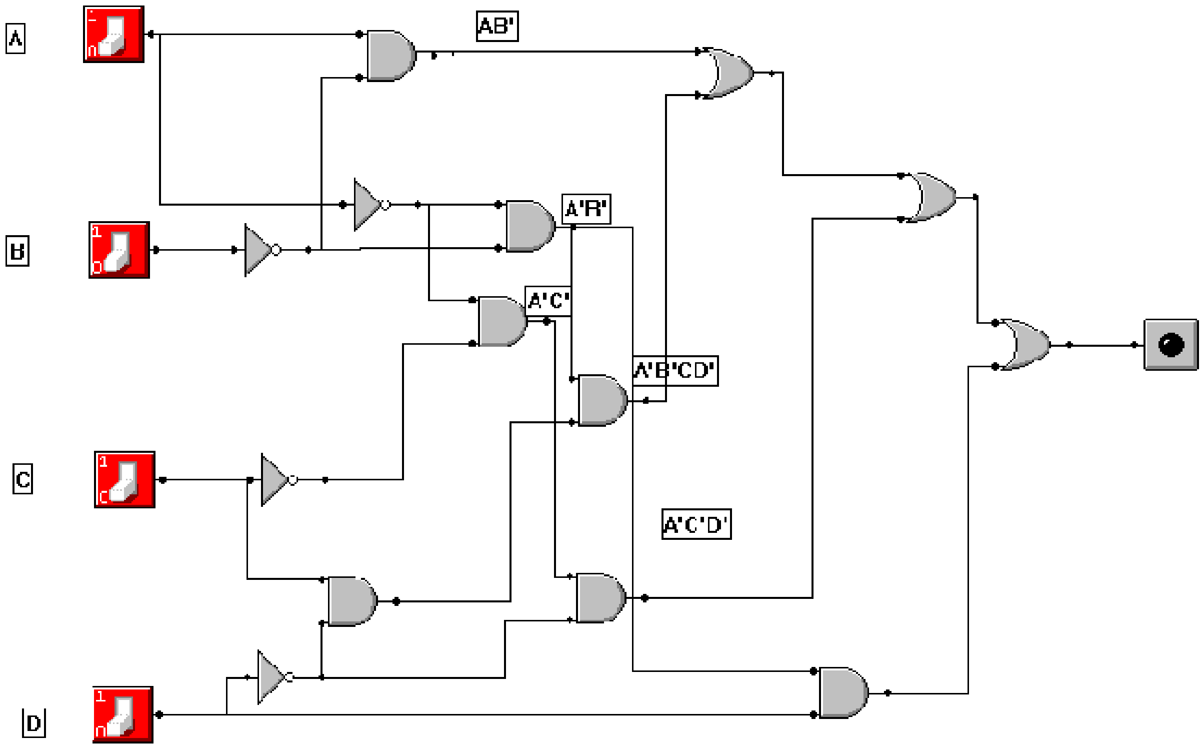


Exercise 10-10

Draw a logic circuit that corresponds to each of the expressions shown below:

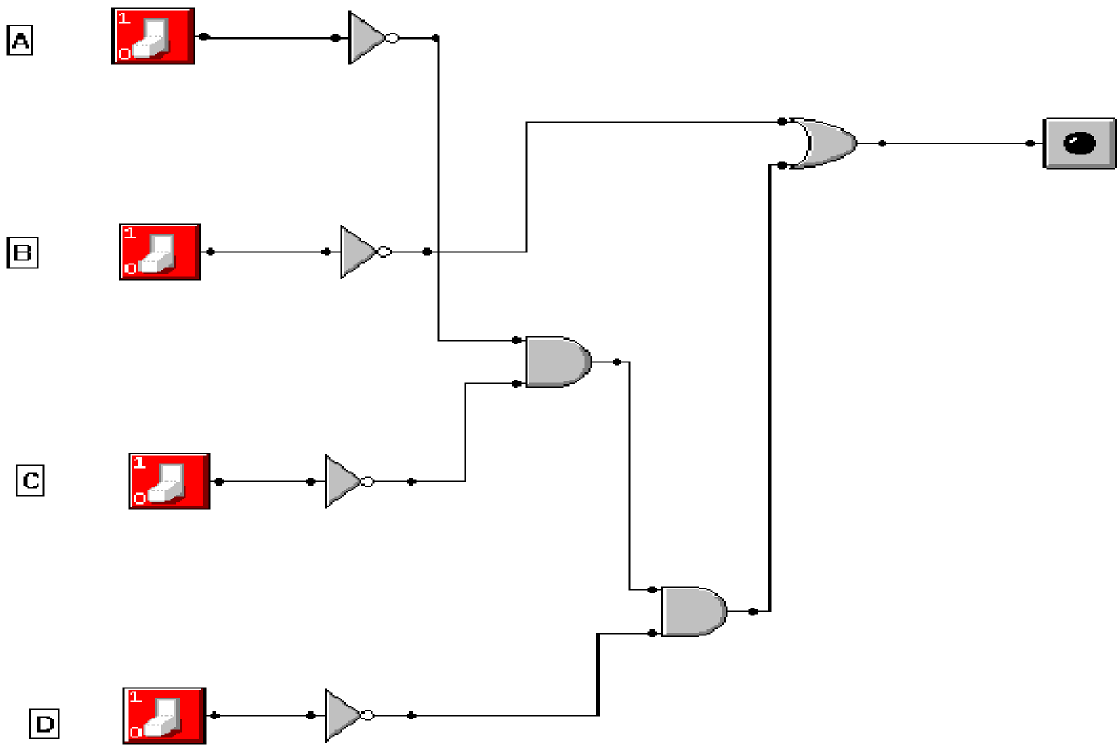
a) $AB' + A'C'D' + A'B'D + A'B'CD'$

Solution:



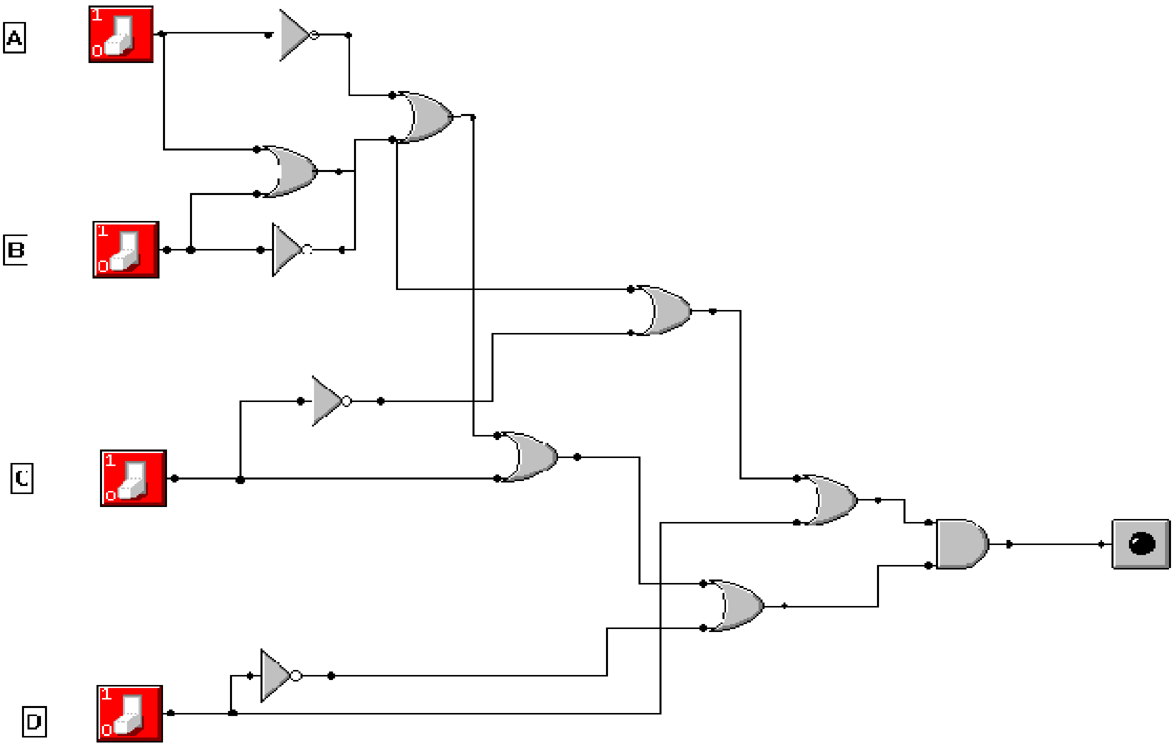
b) $B' + A'C'D'$

Solution:



c) $(A' + B' + C + D')(A + B + C' + D)$

Solution:



Exercise 10-11

Given the following the following truth table, where **A**, **B** and **C** are the input variables and **X** is the output variable.

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

a) Use the sum-of-products algorithm to find the Boolean expression that describes the output of the truth table.

Solution:

$$x = A'B'C' + A'BC + AB'C + ABC'$$

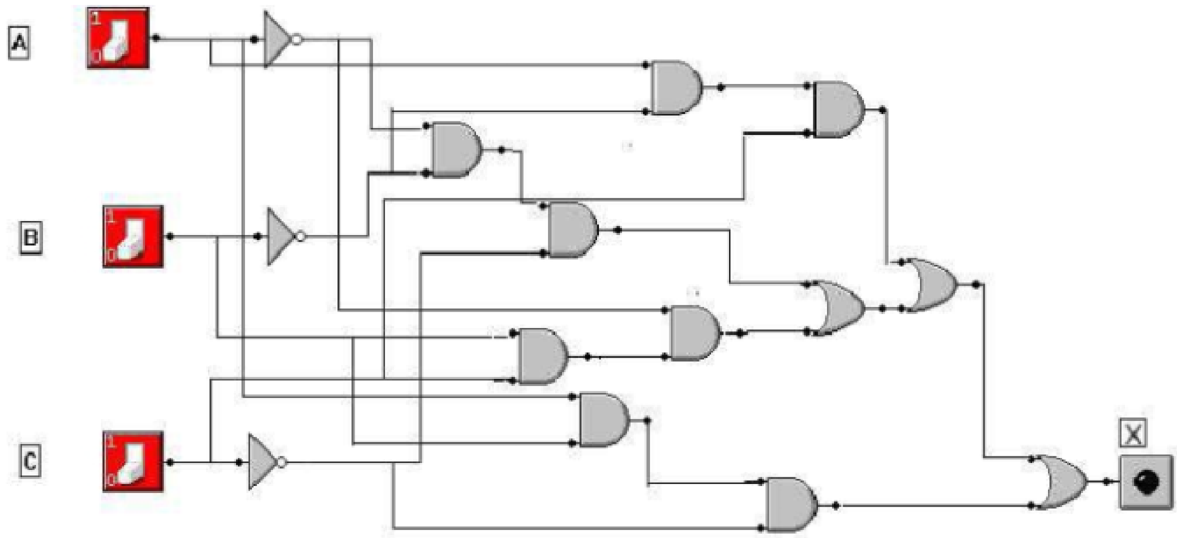
b) What is the functionality of the circuit?

Solution:

The circuit computes the parity of a number. The parity bit is equal to 1 if the number of ones is even otherwise, the parity bit is equal to 0.

c) Draw the Boolean circuit. **Note** that each gate can have only two inputs.

Solution:


A
B
C
X