Abstract

CHIC-2\textsuperscript{1} is an Esprit project on Creating Hybrid solutions for Industry and Commerce. CHIC-2 tackles four large scale combinatorial optimisation problems, each being an application supplied by an end-user partner in the project. In this paper we present the "risk management in energy trading" application. The problem comes from the recent privatisation of electricity in England and Wales. The main goal is to derive an optimal portfolio of contracts (between electricity suppliers and generators) that covers a forecast demand for a period of 12 rolling months such that the financial risks are minimised and the potential profit is maximised. The solution(s) should ideally help a trader in electricity commodity in building profitable portfolios.

We present the iterative mathematical and thinking process we have been through to address this problem from a combinatorial optimisation problem to a decision making problem under uncertainty.

1 Problem definition

The problem is defined as follows. Given:

- Electricity demand and price forecasts for every four hourly period of each day of the year.
- A set of contract profiles with a known loadshape parameterised by its baseload unit, a duration (e.g. one month, six months, one year), and unit price.

minimise the "financial risks" of the company by building and choosing a set of contracts such that the company rules are satisfied. The decision variables are contract volumes. A contract volume commits the baseload unit to a fixed real or integer value.

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The company rules are a set of hard and soft constraints defined by:

- The total volume covered by each selected contract should not exceed 500 GWh
- The total cost of each selected contract should not exceed 15 million pounds.
- The total cover of contracts should remain under the electricity demand curve, and should ideally be as close as possible to the demand. This last requirement defines a set of soft constraints that should be satisfied as closer as possible.

All the constraints are linear. This problem was modelled and solved using the Constraint Logic Programming platform ECL\textsuperscript{IP}S\textsuperscript{e}[4]. The project is currently in its second year. During the first year we focussed mainly on studying the combinatorics of the problem using an iterative prototyping approach. Each iterative step aimed respectively at checking the feasibility of the constraints, experimenting with different cost functions and a multi-criteria approach.

2 The combinatorial search model

Most problems addressed in constraint programming are combinatorial optimisation problems under conditions of certainty. This means that the data are deterministic and fully known whether they are static or dynamic (dependent on time). For such problems the objective criteria to be optimised are usually universally accepted and known.

The risk management problem at hand is ill-defined: the client has only a partial information about the demand and price data, and the decision criteria to control the financial risks are not fully known. However, from the client expertise we could approximate the concept of “financial risk” in terms of demand exposure (sum of all distances between the demand curve and the cover of contracts at any one time), expected profit (sum of all expected profits achieved by selecting and choosing contract volumes as opposed to paying the demand at its forecast price).

We first modelled the problem as a combinatorial optimisation problem, i.e. we assumed that the demand and price forecast data are known with certainty, and that there exists a couple of well defined decision functions: i) minimise the demand exposure, ii) maximise the expected profit of a portfolio of contracts.

After checking the consistency of the constraints using the finite domain constraint solver, we used the CPLEX [2] library of ECL\textsuperscript{IP}S\textsuperscript{e} to find optimal
solutions for each objective criterion. First conclusion; optimising on each function independently solves two different problems:

- Minimising the demand exposure corresponds to a pure bin packing problem, where the objective is to select as many contracts as possible such that the uncovered space is minimised.
- Maximising the expected profit corresponds to a pure knapsack problem where the objective is to select as many contracts as possible such that their profit maximises the total expected profit of the space they fit into.

Then we combined the two objective criteria, by optimising the demand exposure and constraining the expected profit to be within bounds (or vice versa). This simulation showed that the demand exposure and the expected profit work against each other. One reason is that contracts that cover high peaks in demand (thus contribute to minimising the demand exposure) have low expected profit. Indeed, their prices are very high (thus work against maximising the expected profit).

The actual problem seems to contain a combination of both aspects. We could perform many more experiments with different decision criteria and study how they influence the selection of contracts. However, at this stage we believed that a more important revision of our thinking process was necessary. Among the assumptions we have considered so far, some of them set limitations on the nature of the problem:

- Uncertainty. The demand and price data have been considered as fully known. No concept of uncertainty, forecast trends or seasonal pattern have been simulated.
- Risk parameters in decision criteria. The objective criteria do not take any risk aspect into account.
- Heuristics. No preferences for contract selections were defined nor considered during the optimisation process.

These assumptions were left aside because the client did not have a perfect knowledge of the situation but also because we addressed the problem as CLP programmers, that is we assumed the availability of perfect information and we were driven by the idea of solving a large scale combinatorial optimisation problem. This was wrong. The risk management problem is about decision making under uncertainty and should be tackled as such.
3 Dealing with risk and uncertainty

Risk management deals with the uncertainty of the occurrence of an unforeseen event, namely a change in demand or price data, and the impact on the decision making process if the unforeseen event occurs. Since the fifties, such problems have been tackled by mathematical programming tools based on utility theory, decision theory, and game theory but to our knowledge constraint programming has never been used nor contributed to this application domain. In the second year of the project, we now focus on how and what can constraint programming bring to this class of problems.

From a mathematical point of view, there is a distinction between decision making under risk and decision making under uncertainty [6]. Under risk, the data are partially unknown but probability distributions are known or can be secured. When the element of uncertainty is treated as a level of probability, expressed between 0 (impossible) and 1 (certainty), there is no uncertainty anymore. The model becomes a probabilistic model with fixed data and simulations are run to study different probability distributions (e.g. Monte Carlo). Also, one needs to analyze whether the probability distribution is stationary (seasonal trends not taken into account), or non-stationary (probabilities dependent on time). Decision making under risk is usually based on a set of decision criteria like expected profit or other criteria that range from completely risk averse to completely permissive.

Under uncertainty, the situation becomes worth. No probabilistic distributions of the data are available. Our risk management problem faces this situation.

To our knowledge, practical models for decision making under uncertainty are mathematical programming models based on the following four classes of data representation[6].

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Increasing level of mathematical difficulty

Static

Deterministic

Dynamic (time)

Data

Probabilistic

Stationary

Nonstationary (time)
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Raw demand and price data in the risk management application are non-stationary: the data depend on time and may experience seasonal variations
and trends. The risk lies in having a portfolio of contracts that is exposed to any random fluctuation. The complexity of such non-stationary models where no probability distribution is available, is tackled in mathematical programming by means of approximations [6]. The data are either 1) reduced to a certainty case, and thus considered deterministic, 2) attached to randomly generated probability distributions, or 3) transformed into forecast data using forecasting methods.

Each type of approximation corresponds to a mathematical model for decision making under uncertainty: a simple deterministic model, stochastic models, or forecasting models. Deterministic models average the data by their mean and reduce the problem to one under certainty. Stochastic models are based on generating probability distributions and studying the best and worst case scenarios. Forecasting models are designed to detect trends and seasonal variations in past data. The choice among the three models is driven by the nature of the data in order not to remove any effect of seasonal pattern, trend fluctuation, or to ignore any element of risk.

The results we have presented in the previous section correspond the first approximation: decision making under certainty. Even though we have recently introduced a decision criterion that measures risk in demand exposure, our deterministic data representation does not seem to reflect the uncertainty in trends. Concerning the third approximation, our client has had experiences with forecasting methods. These methods have proved to be ideal in predicting trends and seasonal variations for short term planning, but are not reliable for long term planning (one year in our case). Thus we need a new approach for this non-stationary model.

Some experiments based on the second approximation are currently being done by one of our project partners.

4 Further work and research issues

The mathematical model of the risk management application is now being redefined based on three main tasks [3]:

Risk characterisation. The basic elements of risk are i) a degree of uncertainty regarding the occurrence of a change in demand or price data, ii) the (negative) impact on the solution if the problem occurs. The degree of uncertainty is irrelevant if the impact is of poor value (no profit nor loss). Our current research aims at defining a non-stationary model in CLP for the raw data. Ideally, this model should not approximate the uncertainty in data and should take into account the risk impact, or magnitude of potential loss,

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2 Several methods exist to characterise data; they are mainly based on the computation of their mean and variance (measure of the spread, degree of uncertainty)
as a function dependent on time and embedded in the data representation.

*Risk measurement.* Risk can be measured in two ways. The first concerns the expectation of loss called *risk exposure*. It is usually expressed as the product of the risk impact multiplied by the probability, but in our case the probability will be replaced by a dynamic function. This function would express the user expertise as to when the risk impact is relevant or not. The second way, is *risk reduction leverage*. This notion aims at determining the effectiveness of a possible counter measure by comparing costs with its expected profits.

*Risk control.* Controlling risk is to determine actions to reduce risk and possibly make profit, and consequently to define decision criteria. Most decision criteria currently used, describe how conservative is the decision maker with uncertainty conditions. Decision criteria for portfolio selection often try to maximise the expected return for a given risk, minimise the variance, and minimise the risk exposure [5][1]. However, most of these criteria assume probabilistic distributions attached to the data.

**Conclusion** Portfolio selection can be summarized by “choose a portfolio that offers the best trade-off between expected return and risk.” However, existing models are most often based on probabilistic figures (known or arbitrary) and thus generate deterministic data and send uncertainty back to the decision criteria and simulation methods. By exploiting the flexibility of CLP models, we hope to derive a non-stationary model that better fits the uncertainty in data and embeds a representation of impacts if trends and/or fluctuations occur.

**References**


